

# Introduction

Algorithms

# What's an algorithm?

**Algorithm (noun)** – Finite sequence of rigorous instructions, used to solve a problem or to perform a computation [source].

## Al-Khwarizmi

The word **algorithm** comes from the name of a Persian mathematical genius, **Muhammad ibn Musa al-Khwarizmi**. He was born around 780 AD in the region now known as Uzbekistan. He produced vastly influential works in mathematics, astronomy, and geography [[source](#)].



# Algorithm: Core Characteristics

- **Finiteness:** An algorithm must terminate after a finite number of steps.
- **Definiteness:** Each step of the algorithm must be precisely defined and unambiguous.
- **Input:** An algorithm can have zero or more well-defined inputs.
- **Output:** An algorithm produces at least one output.
- **Effectiveness:** Each step must be feasible and executable.

[[source](#)]

# Algorithm: Desirable Characteristics

- **Efficiency:** An algorithm should use minimal resources (time and space).
- **Correctness:** An algorithm must produce the correct output for all valid inputs.
- **Readability:** The algorithm should be easy to understand and follow.
- **Generality:** An algorithm should be applicable to a wide range of inputs.
- **Modularity:** An algorithm can be broken down into smaller, independent subroutines.

[[source](#)]

# Examples of ancient algorithms

- 1600 BC: **Babylonian method** (a.k.a Heron's method) for finding square roots
- 300 BC: **Euclidean algorithm** for finding the greatest common divisor (GCD) of two integers
- 200 BC: **Sieve of Eratosthenes** for finding prime numbers

[\[source\]](#)

# Babylonian Method

Iterative algorithm that starts with an initial guess for the square root and then iteratively refines the estimate by averaging it with the original number divided by the current estimate.

Given a number  $S$ , and an initial guess  $x_0$  for  $S$ , the Babylonian method iterates as:

$$x_{n+1} = \frac{1}{2} \left( x_n + \frac{S}{x_n} \right)$$

The algorithm requires iterating until it converges to a reasonable accurate approximation of the square root of  $S$  [[source](#)].

# Euclidean Algorithm

Algorithm for finding the greatest common divisor (GCD) of two integers.

1. Given two positive integers  $a$  and  $b$ , the algorithm starts by dividing  $a$  by  $b$  and obtaining the remainder  $r$ .
2. If  $r$  is zero, then the algorithm terminates, and the current value of  $b$  is the GCD of  $a$  and  $b$ .
3. If  $r$  is not zero, then the algorithm replaces  $a$  with  $b$  and  $b$  with  $r$ , and then repeats the division step (step 1) with the new values of  $a$  and  $b$ .
4. Repeat steps 1-3 until the remainder  $r$  becomes zero.

[[source](#)]



# Sieve of Eratosthenes

Algorithm for finding all prime numbers up to a specified integer  $n$ . It was devised by the Greek mathematician Eratosthenes of Cyrene.

1. **Create a list of integers:** Start with a list of consecutive integers from 2 to  $n$ .
2. **Initial assumption:** Assume all numbers in the list are prime.
3. **Iterative marking:** Beginning with the first number in the list (2), mark all of its multiples (except itself) as composite (i.e., not prime). Move to the next number in the list that is still assumed to be prime and repeat the process. Continue this until you've processed numbers up to the square root of  $n$ .
4. **Output the primes:** The numbers that remain unmarked in the list are the prime numbers up to  $n$ .

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